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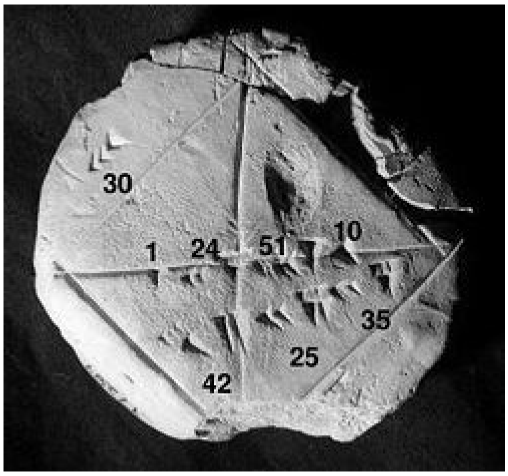
Babylonian mathematics

Babylonian mathematics (also known as *Assyro-Babylonian mathematics*[1][2][3][4][5][6]) was any mathematics developed or practiced by the people of Mesopotamia, from the days of the early Sumerians to the centuries following the fall of Babylon in 539 BC. Babylonian mathematical texts are plentiful and well edited.[7] In respect of time they fall in two distinct groups: one from the Old Babylonian period (1830–1531 BC), the other mainly Seleucid from the last three or four centuries BC. In respect of content, there is scarcely any difference between the two groups of texts. Babylonian mathematics remained constant, in character and content, for nearly two millennia.[7]



Babylonian clay tablet YBC 7289 with annotations. The diagonal displays an approximation of the square root of 2 in four sexagesimal figures, 1 24 51 10, which is good to about six decimal digits. $1 + 24/60 + 51/60^2 + 10/60^3 = 1.41421296...$ The tablet also gives an example where one side of the square is 30, and the resulting diagonal is 42 25 35 or 42.4263888...

In contrast to the scarcity of sources in Egyptian mathematics, knowledge of Babylonian mathematics is derived from some 400 clay tablets unearthed since the 1850s. Written in Cuneiform script, tablets were inscribed while the clay was moist, and baked hard in an oven or by the heat of the sun. The majority of recovered clay tablets date from 1800 to 1600 BC, and cover topics that include fractions, algebra, quadratic and cubic equations and the Pythagorean theorem. The Babylonian tablet YBC 7289 gives an approximation to $\sqrt{2}$ accurate to three significant sexagesimal digits (about six significant decimal digits).

Contents

References

Origins of Babylonian mathematics

Babylonian mathematics is a range of numeric and more advanced mathematical practices in the ancient Near East, written in cuneiform script. Study has historically focused on the Old Babylonian period in the early second millennium BC due to the wealth of data available. There has been debate over the earliest appearance of Babylonian mathematics, with historians suggesting a range of dates between the 5th and 3rd millennia BC.[8] Babylonian mathematics was primarily written on clay tablets in cuneiform script in the Akkadian or Sumerian languages.

"Babylonian mathematics" is perhaps an unhelpful term since the earliest suggested origins date to the use of accounting devices, such as bullae and tokens, in the 5th millennium BC.[9]

Babylonian numerals

The Babylonian system of mathematics was a sexagesimal (base 60) numeral system. From this we derive the modern day usage of 60 seconds in a minute, 60 minutes in an hour, and 360 degrees in a circle.[10] The Babylonians were able to make great advances in mathematics for two reasons. Firstly, the number 60 is a superior highly composite number, having factors of 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 (including those that are themselves composite), facilitating calculations with fractions. Additionally, unlike the Egyptians and Romans, the Babylonians had a true place-value system, where digits written in the left column represented larger values (much as, in our base ten system, 734 = 7×100 + 3×10 + 4×1).[11]

Sumerian mathematics

The ancient Sumerians of Mesopotamia developed a complex system of metrology from 3000 BC. From 2600 BC onwards, the Sumerians wrote multiplication tables on clay tablets and dealt with geometrical exercises and division problems. The earliest traces of the Babylonian numerals also date back to this period.[12]

Old Babylonian mathematics (2000–1600 BC)

Most clay tablets that describe Babylonian mathematics belong to the Old Babylonian, which is why the mathematics of Mesopotamia is commonly known as Babylonian mathematics. Some clay tablets contain mathematical lists and tables, others contain problems and worked solutions.

Arithmetic

The Babylonians used pre-calculated tables to assist with arithmetic. For example, two tablets found at Senkerah on the Euphrates in 1854, dating from 2000 BC, give lists of the squares of numbers up to 59 and the cubes of numbers up to 32. The Babylonians used the lists of squares together with the formulae:

$$ab = \frac{(a+b)^2 - a^2 - b^2}{2}$$

$$ab = \frac{(a+b)^2 - (a-b)^2}{4}$$

to simplify multiplication.

The Babylonians did not have an algorithm for long division.[13] Instead they based their method on the fact that:

$$\frac{a}{b} = a \times \frac{1}{b}$$

together with a table of reciprocals. Numbers whose only prime factors are 2, 3 or 5 (known as 5-smooth or regular numbers) have finite reciprocals in sexagesimal notation, and tables with extensive lists of these reciprocals have been found.

Reciprocals such as 1/7, 1/11, 1/13, etc. do not have finite representations in sexagesimal notation. To compute 1/13 or to divide a number by 13 the Babylonians would use an approximation such as:

$$\frac{1}{13} = \frac{7}{91} = 7 \times \frac{1}{91} \approx 7 \times \frac{1}{90} = 7 \times \frac{40}{3600} = \frac{280}{3600} = \frac{4}{60} + \frac{40}{3600}.$$

Algebra

The Babylonian clay tablet YBC 7289 (c. 1800–1600 BC) gives an approximation of $\sqrt{2}$ in four sexagesimal figures, 1 24 51 10, which is accurate to about six decimal digits,[14] and is the closest possible three-place sexagesimal representation of $\sqrt{2}$:

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3} = \frac{30547}{21600} = 1.41421\overline{296}.$$

As well as arithmetical calculations, Babylonian mathematicians also developed algebraic methods of solving equations. Once again, these were based on pre-calculated tables.

To solve a quadratic equation, the Babylonians essentially used the standard quadratic formula. They considered quadratic equations of the form:

$$x^2 + bx = c$$

where b and c were not necessarily integers, but c was always positive. They knew that a solution to this form of equation is:

$$x = -\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}$$

and they found square roots efficiently using division and averaging.[15] They always used the positive root because this made sense when solving "real" problems. Problems of this type included finding the dimensions of a rectangle given its area and the amount by which the length exceeds the width.

Tables of values of $n^3 + n^2$ were used to solve certain cubic equations. For example, consider the equation:

$$ax^3 + bx^2 = c.$$

Multiplying the equation by a^2 and dividing by b^3 gives:

$$\left(\frac{ax}{b}\right)^3 + \left(\frac{ax}{b}\right)^2 = \frac{ca^2}{b^3}.$$

Substituting $y = ax/b$ gives:

$$y^3 + y^2 = \frac{ca^2}{b^3}$$

which could now be solved by looking up the $n^3 + n^2$ table to find the value closest to the right hand side. The Babylonians accomplished this without algebraic notation, showing a remarkable depth of understanding. However, they did not have a method for solving the general cubic equation.

Growth

Babylonians modeled exponential growth, constrained growth (via a form of sigmoid functions), and doubling time, the latter in the context of interest on loans.

Clay tablets from c. 2000 BCE include the exercise "Given an interest rate of 1/60 per month (no compounding), compute the doubling time." This yields an annual interest rate of 12/60 = 20%, and hence a doubling time of 100% growth/20% growth per year = 5 years.[16][17]

Plimpton 322

The Plimpton 322 tablet contains a list of "Pythagorean triples", i.e., integers (a, b, c) such that $a^2 + b^2 = c^2$. The triples are too many and too large to have been obtained by brute force.

Much has been written on the subject, including some speculation (perhaps anachronistic) as to whether the tablet could have served as an early trigonometrical table. Care must be exercised to see the tablet in terms of methods familiar or accessible to scribes at the time.

> [...] the question "how was the tablet calculated?" does not have to have the same answer as the question "what problems does the tablet set?" The first can be answered most satisfactorily by reciprocal pairs, as first suggested half a century ago, and the second by some sort of right-triangle problems.

(E. Robson, "Neither Sherlock Holmes nor Babylon: a reassessment of Plimpton 322", *Historia Math.* **28** (3), p. 202).

Geometry

Babylonians knew the common rules for measuring volumes and areas. They measured the circumference of a circle as three times the diameter and the area as one-twelfth the square of the circumference, which would be correct if π is estimated as 3. They were aware that this was an approximation, and one Old Babylonian mathematical tablet excavated near Susa in 1936 (dated to

between the 19th and 17th centuries BCE) gives a better approximation of π as 25/8 = 3.125, about 0.5 percent below the exact value.[18] The volume of a cylinder was taken as the product of the base and the height, however, the volume of the frustum of a cone or a square pyramid was incorrectly taken as the product of the height and half the sum of the bases. The Pythagorean theorem was also known to the Babylonians.[19][20][21]

The "Babylonian mile" was a measure of distance equal to about 11.3 km (or about seven modern miles). This measurement for distances eventually was converted to a "time-mile" used for measuring the travel of the Sun, therefore, representing time.[22]

The ancient Babylonians had known of theorems concerning the ratios of the sides of similar triangles for many centuries, but they lacked the concept of an angle measure and consequently, studied the sides of triangles instead.[23]

The Babylonian astronomers kept detailed records of the rising and setting of stars, the motion of the planets, and the solar and lunar eclipses, all of which required familiarity with angular distances measured on the celestial sphere.[24]

They also used a form of Fourier analysis to compute ephemeris (tables of astronomical positions), which was discovered in the 1950s by Otto Neugebauer.[25][26][27][28] To make calculations of the movements of celestial bodies, the Babylonians used basic arithmetic and a coordinate system based on the ecliptic, the part of the heavens that the sun and planets travel through.

Tablets kept in the British Museum provide evidence that the Babylonians even went so far as to have a concept of objects in an abstract mathematical space. The tablets date from between 350 and 50 B.C.E., revealing that the Babylonians understood and used geometry even earlier than previously thought. The Babylonians used a method for estimating the area under a curve by drawing a trapezoid underneath, a technique previously believed to have originated in 14th century Europe. This method of estimation allowed them to, for example, find the distance Jupiter had traveled in a certain amount of time.[29]

Influence

Since the rediscovery of the Babylonian civilization, it has become apparent that Greek and Hellenistic mathematicians and astronomers, and in particular Hipparchus, borrowed greatly from the Babylonians.

Franz Xaver Kugler demonstrated in his book *Die Babylonische Mondrechnung* ("*The Babylonian lunar computation*", Freiburg im Breisgau, 1900) the following: Ptolemy had stated in his *Almagest* IV.2 that Hipparchus improved the values for the Moon's periods known to him from "even more ancient astronomers" by comparing eclipse observations made earlier by "the Chaldeans", and by himself. However, Kugler found that the periods that Ptolemy attributes to Hipparchus had already been used in Babylonian ephemerides, specifically the collection of texts nowadays called "System B" (sometimes attributed to Kidinnu). Apparently, Hipparchus only confirmed the validity of the periods he learned from the Chaldeans by his newer observations.

It is clear that Hipparchus (and Ptolemy after him) had an essentially complete list of eclipse observations covering many centuries. Most likely these had been compiled from the "diary" tablets: these are clay tablets recording all relevant observations that the Chaldeans routinely made. Preserved

examples date from 652 BC to AD 130, but probably the records went back as far as the reign of the Babylonian king Nabonassar: Ptolemy starts his chronology with the first day in the Egyptian calendar of the first year of Nabonassar, i.e., 26 February 747 BC.

This raw material by itself must have been hard to use, and no doubt the Chaldeans themselves compiled extracts of e.g., all observed eclipses (some tablets with a list of all eclipses in a period of time covering a saros have been found). This allowed them to recognise periodic recurrences of events. Among others they used in System B (cf. *Almagest* IV.2):

- 223 synodic months = 239 returns in anomaly (anomalistic month) = 242 returns in latitude (draconic month). This is now known as the saros period, which is useful for predicting eclipses.
- 251 (synodic) months = 269 returns in anomaly
- 5458 (synodic) months = 5923 returns in latitude
- 1 synodic month = 29;31:50:08:20 days (sexagesimal; 29.53059413… days in decimals = 29 days 12 hours 44 min 3⅓ s, P.S. real time is 2.9 s, so 0.43 seconds off)

The Babylonians expressed all periods in synodic months, probably because they used a lunisolar calendar. Various relations with yearly phenomena led to different values for the length of the year.

Similarly, various relations between the periods of the planets were known. The relations that Ptolemy attributes to Hipparchus in *Almagest* IX.3 had all already been used in predictions found on Babylonian clay tablets.

All this knowledge was transferred to the Greeks probably shortly after the conquest by Alexander the Great (331 BC). According to the late classical philosopher Simplicius (early 6th century AD), Alexander ordered the translation of the historical astronomical records under supervision of his chronicler Callisthenes of Olynthus, who sent it to his uncle Aristotle. Although Simplicius is a very late source, his account may be reliable. He spent some time in exile at the Sassanid (Persian) court and may have accessed sources otherwise lost in the West. It is striking that he mentions the title *tèresis* (Greek: guard), which is an odd name for a historical work, but is an adequate translation of the Babylonian title *MassArt* meaning *guarding*, but also *observing*. Anyway, Aristotle's pupil Callippus of Cyzicus introduced his 76-year cycle, which improved on the 19-year Metonic cycle, about that time. He had the first year of his first cycle start at the summer solstice of 28 June 330 BC (Proleptic Julian calendar date), but later he seems to have counted lunar months from the first month after Alexander's decisive battle at Gaugamela in fall 331 BC. So Callippus may have obtained his data from Babylonian sources and his calendar may have been anticipated by Kidinnu. Also it is known that the Babylonian priest known as Berossus wrote around 281 BC a book in Greek on the (rather mythological) history of Babylonia, the *Babyloniaca*, for the new ruler Antiochus I; it is said that later he founded a school of astrology on the Greek island of Kos. Another candidate for teaching the Greeks about Babylonian astronomy/astrology was Sudines who was at the court of Attalus I Soter late in the 3rd century BC.

In any case, the translation of the astronomical records required profound knowledge of the cuneiform script, the language, and the procedures, so it seems likely that it was done by some unidentified Chaldeans. Now, the Babylonians dated their observations in their lunisolar calendar, in which months and years have varying lengths (29 or 30 days; 12 or 13 months respectively). At the time they did not use a regular calendar (such as based on the Metonic cycle like they did later) but started a new month based on observations of the New Moon. This made it very tedious to compute the time interval between events.

What Hipparchus may have done is transform these records to the Egyptian calendar, which uses a fixed year of always 365 days (consisting of 12 months of 30 days and 5 extra days): this makes computing time intervals much easier. Ptolemy dated all observations in this calendar. He also writes that "All that he (=Hipparchus) did was to make a compilation of the planetary observations arranged in a more useful way" (*Almagest* IX.2). Pliny states (*Naturalis Historia* II.IX(53)) on eclipse predictions: "After their time (=Thales) the courses of both stars (=Sun and Moon) for 600 years were prophesied by Hipparchus, …". This seems to imply that Hipparchus predicted eclipses for a period of 600 years, but considering the enormous amount of computation required, this is very unlikely. Rather, Hipparchus would have made a list of all eclipses from Nabonasser's time to his own.

Other traces of Babylonian practice in Hipparchus' work are:

- first known Greek use of the division the circle in 360 degrees of 60 arc minutes.
- first consistent use of the sexagesimal number system.
- the use of the unit *pechus* ("cubit") of about 2° or 2½°.
- use of a short period of 248 days = 9 anomalistic months.

See also

- Babylonia
- Babylonian astronomy
- History of mathematics
- Islamic mathematics for mathematics in Islamic Iraq/Mesopotamia

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20. Høyrup, p. 406. "*To judge from this evidence alone* it is therefore likely that the Pythagorean rule was discovered within the lay surveyors' environment, possibly as a spin-off from the problem treated in Db_2-146, somewhere between 2300 and 1825 BC." (Db_2-146 is an Old Babylonian clay tablet from Eshnunna concerning the computation of the sides of a rectangle given its area and diagonal.)

21. Robson 2008, p. 109. "Many Old Babylonian mathematical practitioners … knew that the square on the diagonal of a right triangle had the same area as the sum of the squares on the length and width: that relationship is used in the worked solutions to word problems on cut-and-paste 'algebra' on seven different tablets, from Ešnuna, Sippar, Susa, and an unknown location in southern Babylonia."
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